5/11/18 "Eighty percent of success is showing up." -Woody Allen

HW: "Sinusoidal Modeling" homework section
Test 2 on Wednesday 5/30

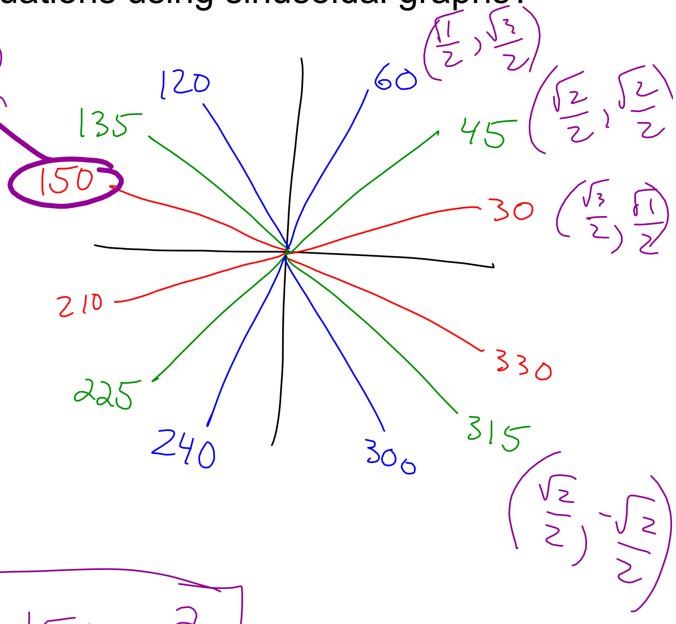
AIM: How do we model real world situations using sinusoidal graphs?

Warm Up:

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ cos sin

Find the exact value of csc 150^{o}

(use unit circle)



csc is reciprocal of sin

$\sin 150 = \frac{1}{2}$

$\boxed{\csc 150 = \frac{2}{1}}$

Name: Answer Key Date: ____________

VERTICAL SHIFTING OF SINUSOIDAL GRAPHS
COMMON CORE ALGEBRA II HOMEWORK

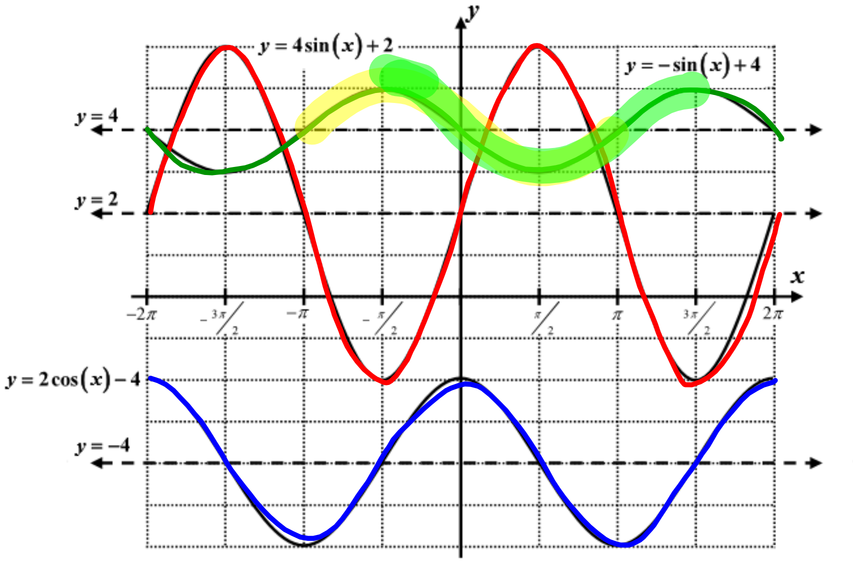
FLUENCY

1. Sketch each of the following equations on the graph grid below. Label each with its equation.

$y = 4\sin(x) + 2$

$y = 2\cos(x) - 4$

$y = -\sin(x) + 4$



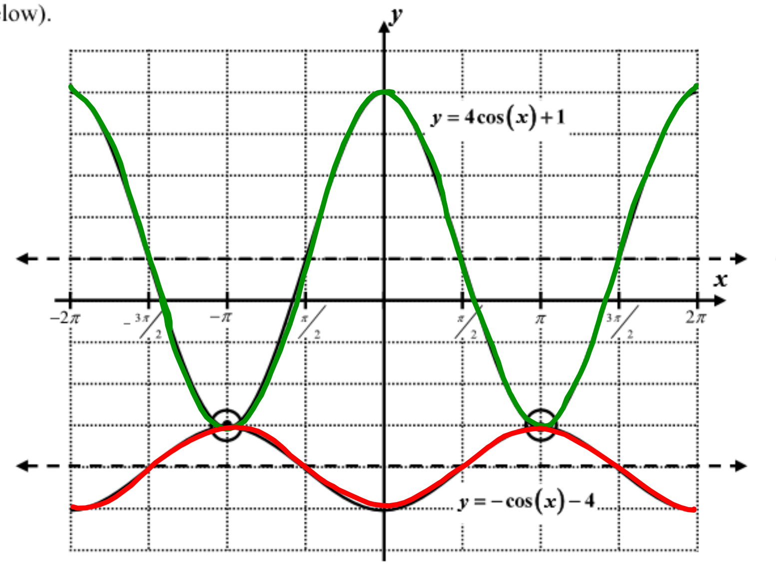
2. Graph and label both of the curves below. Then, state their intersection points (in other words, solve the system of equations shown below).

$y = 4\cos(x) + 1$

$y = -\cos(x) - 4$

Intersection Points:

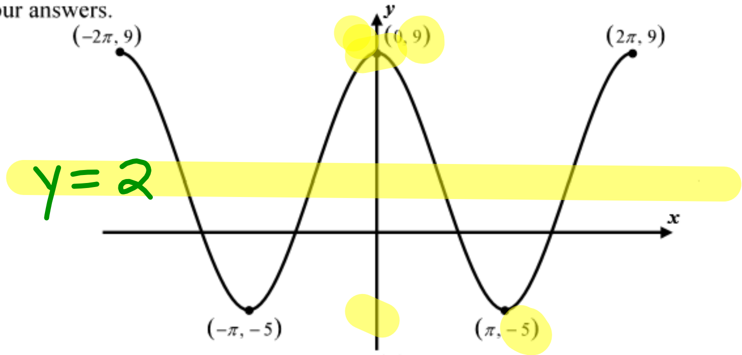
$\boxed{(-\pi, -3) \text{ and } (\pi, -3)}$



4. The following graph can be described using an equation of the form $y = A\cos(x) + C$. Determine the values of A and C. Show how you arrived at your answers.

$C = \frac{-5+9}{2} = \frac{4}{2} = 2$

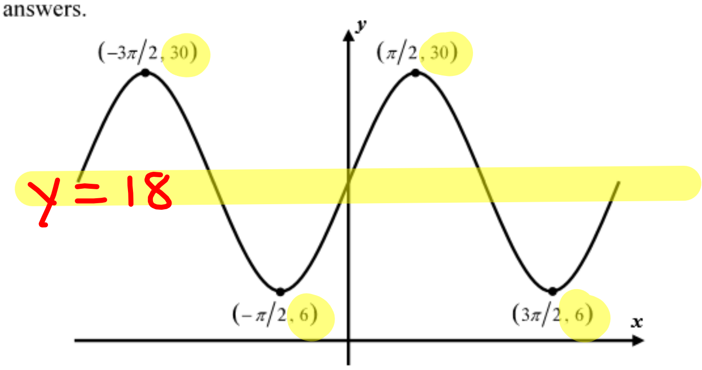
$A = \frac{9-(-5)}{2} = \frac{14}{2} = 7$



5. The following graph can be described using an equation of the form $y = A\sin(x) + C$. Determine the values of A and C. Show how you arrived at your answers.

$C = \frac{6+30}{2} = \frac{36}{2} = 18$

$A = \frac{30-6}{2} = \frac{24}{2} = 12$



6. State the range of each of the following sinusoidal functions in interval form.

(a) $y = 10\sin(x) - 3$

$y_{max} = -3 + 10 = 7$, $y_{min} = -3 - 10 = -13$

Range: $[-13, 7]$

(b) $y = -8\cos(x) + 2$

$y_{max} = 2 + 8 = 10$, $y_{min} = 2 - 8 = -6$

Range: $[-6, 10]$

(c) $y = 22\sin(x) + 30$

$y_{max} = 30 + 22 = 52$, $y_{min} = 30 - 22 = 8$

Range: $[8, 52]$

7. When graphed, the line $y = 14$ would not intersect the graph of which of the following functions?

~~(1)~~ $y = 5\cos(x) + 9$ ~~(3)~~ $y = 2\sin(x) + 15$

~~(2)~~ $y = -6\cos(x) + 10$ (4) $y = 3\sin(x) + 20$

For the curve $y = 3\sin(x) + 20$ the range is $[17, 23]$, thus the graph would never intersect the horizontal line $y = 14$.

(4)

8. Which of the following functions has a maximum value of 25?

(1) $y = 25\sin(x) + 12$ (3) $y = 8\cos(x) + 17$

(2) $y = -10\cos(x) + 35$ (4) $y = 5\sin(x) + 15$

For choice (3): $y_{max} = 17 + 8 = 25$

(3)

SINUSOIDAL MODEL COEFFICIENTS

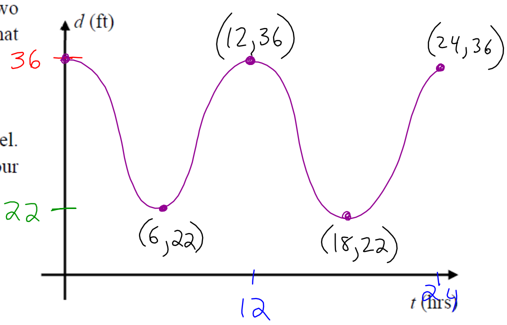
For $y = A\sin(Bx)+C$ and $y = A\cos(Bx)+C$

$\lvert A \rvert$	the **amplitude** or distance the sinusoidal model rises and falls above its midline
C	the **midline** or average y-value of the sinusoidal model
B	the **frequency** of the sinusoidal model – related to the **period**, P, by the equation $BP = 2\pi$
P	the **period** of the sinusoidal model – the minimum distance along the x-axis for the cycle to repeat

Exercise #1: The tides in a particular bay can be modeled with an equation of the form $d = A\cos(Bt)+C$, where t represents the number of hours since high-tide and d represents the depth of water in the bay. The maximum depth of water is 36 feet, the minimum depth is 22 feet and high-tide is hit every 12 hours.

Max Min Period

(a) On the axes, sketch a graph of this scenario for two full periods. Label the points on this curve that represent high and low tide.



(b) Determine the values of A, B, and C in the model. Verify your answers and sketch are correct on your calculator.

$A = \frac{36-22}{2} = \frac{14}{2} = \boxed{7}$

$C = \frac{36+22}{2} = \frac{58}{2} = \boxed{29}$

$B = B \cdot \text{Period} = 2\pi$

$B \cdot 12 = 2\pi$

$B = \frac{2\pi}{12} = \frac{\pi}{6}$

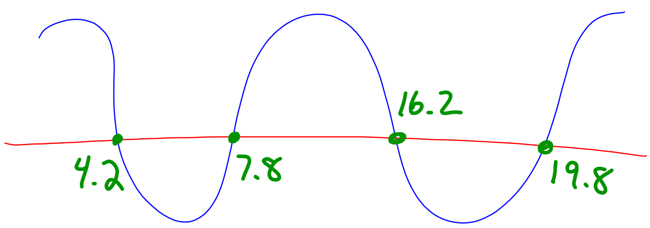
$B = \frac{\pi}{6}$

Equation

$\text{Depth} = 7\cos\left(\frac{\pi}{6}t\right)+29$

(c) Tanker boats cannot be in the bay when the depth of water is less or equal to 25 feet. Set up an inequality and solve it graphically to determine all points in time, t, on the interval $0 \le t \le 24$ when tankers cannot be in the bay. Round all times to the nearest *tenth* of an hour.

$25 \ge 7\cos\left(\frac{\pi}{6}t\right)+29$



$[4.2, 7.8]$ $[16.2, 19.8]$

***Exercise* #2:** The height of a yo-yo above the ground can be well modeled using the equation $h = 1.75\cos(\pi t) + 2.25$, where h represents the height of the yo-yo in feet above the ground and t represents time in seconds since the yo-yo was first dropped from its maximum height.

(a) Determine the maximum and minimum heights that the yo-yo reaches above the ground. Show the calculations that lead to your answers.

Max = 2.25 + 1.75 = 4 ft

Min = 2.25 − 1.75 = .5 ft

(b) How much time does it take for the yo-yo to return to the maximum height for the first time?

Period

freq = π

$2\pi = (\text{freq})(\text{Period})$

$2\pi = \pi \cdot p$

$\boxed{\text{2 seconds} = p}$

***Exercise* #3:** A Ferris wheel is constructed such that a person gets on the wheel at its lowest point, five feet above the ground, and reaches its highest point at 130 feet above the ground. The amount of time it takes to complete one full rotation is equal to 8 minutes. A person's vertical position, y, can be modeled as a function of time in minutes since they boarded, t, by the equation $y = A\cos(Bt) + C$. Sketch a graph of a person's vertical position for one cycle and then determine the values of A, B, and C. Show the work needed to arrive at your answers.

$A = \frac{130-5}{2} = 62.5$

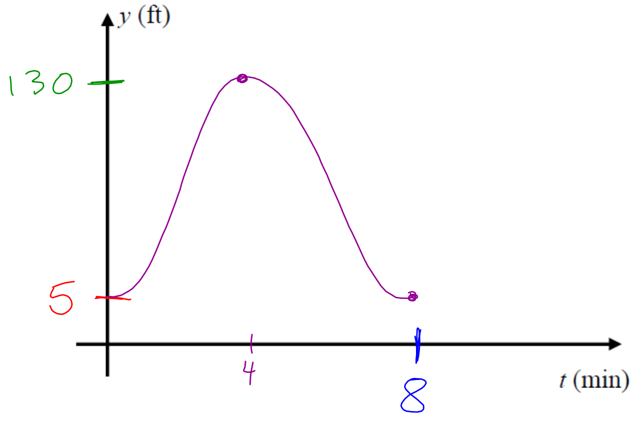
$C = \frac{130+5}{2} = 67.5$

$\boxed{B = \frac{\pi}{4}}$

freq · Period = 2π

$B \cdot 8 = 2\pi$

$B = \frac{2\pi}{8} = \frac{\pi}{4}$



***Exercise* #4:** The possible hours of daylight in a given day is a function of the day of the year. In Poughkeepsie, New York, the minimum hours of daylight (occurring on the Winter solstice) is equal to 9 hours and the maximum hours of daylight (occurring on the Summer solstice) is equal to 15 hours. If the hours of daylight can be modeled using a sinusoidal equation, what is the equation's amplitude?

(1) 6 (3) 3

(2) 12 (4) 4

$A = \frac{15-9}{2} = \frac{6}{2} = 3$